Calc III , August 27

§12.2 Vectors

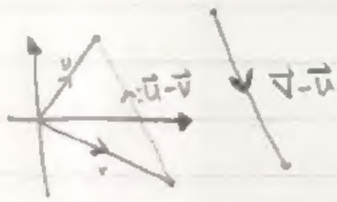
linear shifts of vectors are equivalent, "same vector"
, ie same "direction & magnitude"

$|\vec{v}|$: magnitude of $\vec{v}$
easily found by d formula

Direction of Vectors?

Vector Operations:

1 · taking magnitude $\vec{v} \mapsto \mathbb{R} \geq 0$
2 · adding vectors $\vec{v}_1 \mapsto \vec{v}$ (tip to tail)
3 · subtracting vectors $\vec{v}_3 \mapsto \vec{v}$ (tip to tip)
4 · negating vector $\vec{v} \mapsto \vec{v}$
5 · scalar multiplication $s \cdot \vec{v} \mapsto \vec{v}$



Components of Vectors

every vector has unique representative line segment with tail at origin
↳ given that vector from origin, head coords determine the vector ie $\langle x, y \rangle$

vector  has components $t - s = \langle x, y \rangle$

Algebraic Operations (in 3-space)

1) Magnitude

$$\vec{v} = \langle v_1, v_2, v_3 \rangle$$
$$|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$ ⊂ distance formula

2) Addition

$$\vec{u} = \langle u_1, u_2, u_3 \rangle \quad \vec{v} = \langle v_1, v_2, v_3 \rangle$$
$$\vec{u} + \vec{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$$

3) Subtraction

~(also component-wise)
(same as addition)~

4) Negation

$\vec{u} = \langle u_1, u_2, u_3 \rangle$

$-\vec{u} = \langle -u_1, -u_2, -u_3 \rangle$

5) Scalar Multiplication

$c \in \mathbb{R} \quad \vec{u} = \langle u_1, u_2, u_3 \rangle$

$c\vec{u} = \langle cu_1, cu_2, cu_3 \rangle$

~~Properties~~ of Vector Operations

Let $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^n$ & $a, b \in \mathbb{R}$

1) $(\vec{u}+\vec{v})+\vec{w} = \vec{u}+(\vec{v}+\vec{w})$ (associativity)
2) $\vec{u}+\vec{v} = \vec{v}+\vec{u}$ (commutativity)
3) $\vec{0}+\vec{v} = \vec{v}$ (identity)
4) $\vec{u}+(-\vec{u}) = 0$ (negatives exist)
5) $a(b\vec{u}) = ab(\vec{u})$
6) $a(\vec{u}) + b(\vec{u}) = (a+b)\vec{u}$
7) $a(\vec{u}) + a(\vec{v}) = a(\vec{u}+\vec{v})$
8) $1(\vec{u}) = \vec{u}$ and $0\vec{u} = \vec{0}$

Direction

Property of magnitude $c \in \mathbb{R} \quad \vec{u} \in \mathbb{R}^n$

$|c\vec{u}| = |c| \cdot |\vec{u}|$

Def: direction of $\vec{u}$ = associated unit vector (v with length 1)

$= \frac{1}{|\vec{u}|} \cdot \vec{u}$ when $\vec{u} \neq 0$

Claim $\frac{1}{|\vec{u}|}\vec{u}$ is unit vector

$$=\left|\frac{1}{|\vec{u}|}\vec{u}\right| = \frac{1}{|\vec{u}|}|\vec{u}| = 1$$

Component Vectors

$$\left.\begin{aligned}\vec{i} &= \langle 1,0,0\rangle \\ \vec{j} &= \langle 0,1,0\rangle \\ \vec{k} &= \langle 0,0,1\rangle\end{aligned}\right] \text{standard basis for } \mathbb{R}^3$$